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**Predicting the Initial Crack Length in a Solid Propellant**

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**Abstract**

In this study, numerical analyses were performed to predict the initial crack length of specimens with two different sizes of holes using multi-level simulation techniques. The specimens were subjected to a constant strain rate of 0.33 in/in/min at room temperature. The criterion for determining the initial crack length was based on the instability of the material near the edge of the hole. The results of the analyses show that the predicted initial crack lengths compared well with the experimental measurements.

**Introduction**

In this study, a micro-macromechanical approach was used to predict the initial crack length near the edge of the hole in solid propellant specimens. The approach was based on a simplified micromechanical model, damage mechanics at the micro-level, and finite element analysis at the macro-level (1-3). Both micromechanical and macromechanical analyses were conducted in tandem. The developed technique, together with a mechanistic criterion, was used to predict the initial crack length in high stress regions. The criterion was based on the instability of the damaged material just ahead of the crack tip. The initial crack length is equal to the length of unstable material zone when the damage at the crack tip element is saturated. Based on the definition of the initial crack length and the micro-macromechanical approach, the initial crack lengths in the high stress regions were predicted. The predicted initial crack lengths and the experimentally measured values were compared and the results are discussed.

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## Numerical Modeling

The computer modeling and simulation technique used in this study is called a micro/macro-approach. The approach is similar to the local-global approach in structural analysis. The micro/macro-approach utilizes the two levels of analyses: micro-analysis and macro-analysis. The micro-analysis is performed at the constituent material level, like the reinforcing (particle) material and the binding matrix material, while the macro-analysis is undertaken at the composite specimen (or structural) level. The two analyses are conducted in tandem as damage initiates and evolves in composite specimens (structures). Damage is described at the micro-level in terms of the constituent materials. Damage modes for a particulate composite are described in terms of particle cracking, matrix micro cracking, and interface debonding. The interaction between the two levels of analyses is shown in Fig 1 and the description of each analysis is given below.

### Macro-Analysis

The macro-analysis is conducted at the composite specimen (structure) level. The specimen considered in this study is shown in Fig. 2. It is a square plate with a circular hole at the center. The specimen geometry is 76.2 mm x 76.2 mm and 6.35 mm thick. The radii of the hole are 3.17 mm and 6.35 mm. The finite element analysis technique is used for the macro-analysis. Three-dimensional solid elements were used for the specimen. Because of symmetry, one-eighth of the specimen was modeled. A refined mesh was used around the notch tip to capture local damage and the initial crack.

Isoparametric types of elements are used with the Gauss quadrature rule. These elements require material properties at the Gauss quadrature points. As a result, the interaction between the micro-analysis and the macro-level finite element analysis is performed at the Gauss quadrature points. The micro-analysis provides the Gauss quadrature points of the macro-analysis with effective composite materials properties including the damage state. The macro-analysis is undertaken to compute deformation and stress/strain of the composite specimen (structure). These data are transferred to the micro-level analysis.

### Micro-Analysis

Because the micro-analysis is conducted repeatedly at each Gauss quadrature point of finite elements as damage progresses, the analysis should be computationally efficient. To this end, a simplified analytical model, or a unit-cell model, is used for the micro-level analysis.

The micro-analysis has two major functions. One is to compute effective material properties of the composite based on the progressive damage at the constituent material level. These data are used for the subsequent macro-analysis. The other

\* Suggest spelling out to match format of figures: Fig. = Figure (throughout paper)

# = space needed

function is to decompose the macro-level stress/strain obtained from the previous macro-analysis into micro-level stress/strain. The micro-stress/strain is used to determine the damage state at the constituent material level. The continuum damage theory is utilized to describe the damage state as well as the degraded material properties of each constituent material. The micro-analysis utilizes the degraded material properties to calculate the updated material properties of the damaged composite.

### Continuum Damage Theory

Damage at the micro-level is described using a continuum damage theory. In general, an anisotropic damage theory can be applied to the constituent materials because the damage state may be anisotropic depending on the given conditions. An anisotropic damage theory, however, requires many material parameters, most of which are not easily obtainable. On the other hand, a simpler isotropic damage theory may be used at the micro-level, assuming that each constituent material behavior is isotropic at the level and its damage state is also isotropic. These assumptions are realistic. Therefore, an isotropic damage theory is used in this study. The particulate composite under consideration has matrix damage, including interface debonding but not particle cracking. As a result, the discussion below applies to the matrix material.

For an isotropic damage theory, a scalar damage parameter is used to indicate the degree of damage in the matrix material. The damage parameter  $d$  varies from 0 (denoting no damage) to  $d_c < 1$  (denoting damage saturation). When damage saturates in the material, the stress in the material reduces to zero because the material cannot support the external load. The damaged material has reduced stiffness as expressed below:

$$E_{ijkl}^r = E_{ijkl}^o (1 - d) \quad (1)$$

in which superscripts ' $r$ ' and ' $o$ ' denote reduced and original moduli, respectively.

The damage evolution function for the damage parameter is described based on an experimental observation. The experimental study showed that damage increases as the hydrostatic stress increases. Therefore, the dilatational strain energy was an important part in the damage process. Previous studies used the total strain energy for the damage evolution function, and their results were in good agreement with the experimental data. In the present study, the damage evolution function is assumed to be a function of dilatational strain energy of the matrix material. Under a uniaxial load, which results in a nearly uniaxial state of stress near the notch tip, either using the total strain energy or the dilatational strain energy for the damage function may not make much difference because both functions are proportional to the square of the uniaxial stress component. However, for a complex loading, the total strain energy would be a better choice. In this study, the rate of damage increase,  $\dot{d}$ , is assumed to

be proportional to the rate of dilatational strain energy increase,  $\dot{U}_d$ , if the present dilatational strain energy exceeds the threshold value  $U_d^0$ .

$$\dot{d} \propto \dot{U}_d \text{ if } U_d > U_d^0 \quad (2)$$

The proportional constant and the threshold dilatational strain energy are the material properties of the matrix. The damage model presented here includes both matrix cracking and interface cracking at the micro-level. However, if the interface damage is modeled separately, the following approach can be used, but it was not used for the results presented in this paper. One of the reasons the interface damage was included in the matrix damage is that interface strength data is not available with any associated failure criterion. However, separate modeling of matrix damage and interface debonding would result in a more detailed failure process.

For particle/matrix interface damage, it is assumed that the damage growth at the interface is related to the change of the volume ~~like~~ :

$$d\phi = (1-\phi)d\varepsilon_u \quad (3)$$

Where  $\phi$  is the interface damage parameter and  $\varepsilon_u$  is the strain dilatation. This equation applies when the induced strain measure exceeds the threshold value that is material dependent.

### Criterion for Initial Crack Length

As the external load (uniform displacement in this case) is applied to the specimen, as shown in Fig. 2, there is a stress/strain rise near the notch tip (see curve #1 in Fig. 3a) before damage occurs. The curve #1 in Fig. 3a plots the characteristic, induced stress distribution along the minimum section across the hole of the specimen before damage initiation. Damage initiates ~~first~~ at the notch tip and in its immediate neighborhood, and ~~it~~ grows there faster than at other locations of the specimen. Once damage begins to accumulate at the notch tip zone, the material in the zone becomes softer, as described. The load-bearing capability of the softer material zone reduces by transferring its burden to the neighboring material zone with less damage (i.e. stiffer material). Therefore, the induced stress distribution along the minimum section is represented in curve #2 of Fig. 3a as damage grows at the notch tip zone. Further damage growth eventually results in a lower stress at the zone very close to the notch tip than at its neighboring zone, as seen in curve #3 of Fig. 3a. Finally, as damage saturates at the notch tip, the induced stress vanishes there, as shown in Fig. 3b. No load-carrying capability at the notch tip indicates a crack formation at the notch tip at the given load. Then, the main question is how far the crack will propagate to be the initial crack at the given load.

? Wording is a bit confusing - suggest rewriting

In order to determine the initial crack size, the unstable (softening) material zone is determined. The unstable material zone means the induced stress decreases even if the applied strain (displacement) increases. Once the crack forms at the notch tip, it is considered to propagate through the unstable material zone until it meets the stable material zone. For example, Fig. 3b illustrates both unstable and stable material zones. In summary, the length of <sup>the</sup> initial crack is assumed to be the same as the unstable (softening) material zone when damage saturates at the notch tip (i.e. the stress at the notch tip vanishes).

## Results

Figure 4 shows the stress-strain <sup>should be superscripted</sup> curves for the uncracked specimen tested at a constant strain rate of 6.67 min<sup>-1</sup>. Also included is the predicted stress-strain curve based on the finite element analysis. It is seen that the stress-strain curves obtained from test and numerical analysis agree <sup>very</sup> well. Similarly, for the cracked specimen, there is a good correlation exists <sup>ing</sup> between the measured and the predicted stress-strain curves. Figures 5 and 6 show the distributions of the normal stress as a function of the normalized distance from the edge of the hole. According to these two figures, together with the criterion for the determination of the initial crack length, the predicted initial crack lengths are 0.0489 in. and 0.0242 in. for the small ( $r = 0.125$  in.) and the large size <sup>d</sup> ( $r = 0.25$  in.) hole<sup>s</sup>, respectively. The measured averaged values of the initial crack length are 0.0307 in. and 0.0526 in. for the small and the large size <sup>d</sup> holes, respectively. The good correlation between the predicted and the measured values of the initial crack length indicates that the proposed method can be used to predict the initial crack length with good accuracy.

if going to use "averaged" you need a comma in front. If using "average values" get rid of "d"

## Conclusions

In this study, a micro-macro approach was used for the prediction of initial crack length initiating from the edge of the hole under a constant strain rate condition. The unstable material zone caused by large damage was defined and used <sup>as</sup> for the criterion for the initial crack size. The predicted results agreed well with the experimental measurements.

"...measured, averaged values..."  
or

"...measured average values..."

## References

- (1) Kwon, Y.W., Lee, J.H., and Liu, C.T., "Modeling and Simulation of Crack Initiation and Growth in Particulate Composites," Journal of Pressure Vessel technology, Vol. 119, August 1997
- (2) Kwon, Y.W. and Liu, C.T., "Study of Damage Evolution in Composites Using Damage Mechanics and Micromechanics," Composite Structures, Vol. 38, No. 1-4, 1997.

Quotation marks need to be next to title and facing title (cc)

- (3) Kwon, Y. W. and Baron, D.T., "Numerical Predictions of Progressive Damage Evolution in particulate Composites," Journal of Reinforced Plastics and Composites, Vol. 17, No. 8, 1998.

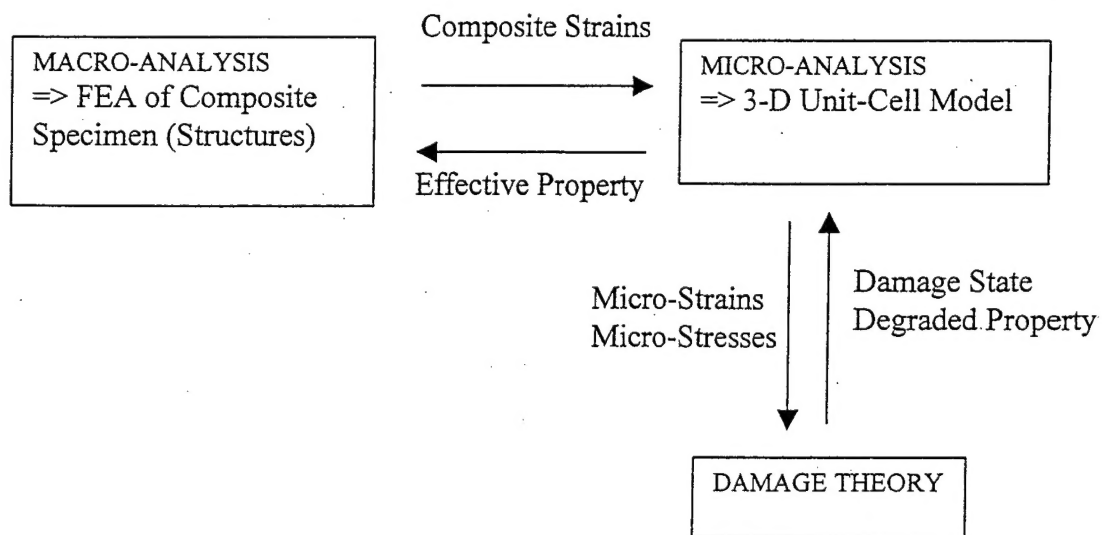


Figure 1. Interaction between Micro-analysis and Macro-analysis



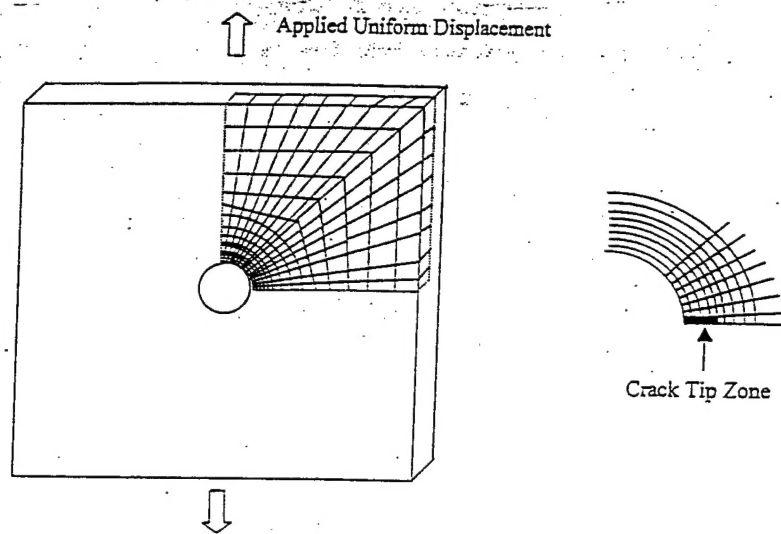


Figure 2. Finite Element Model

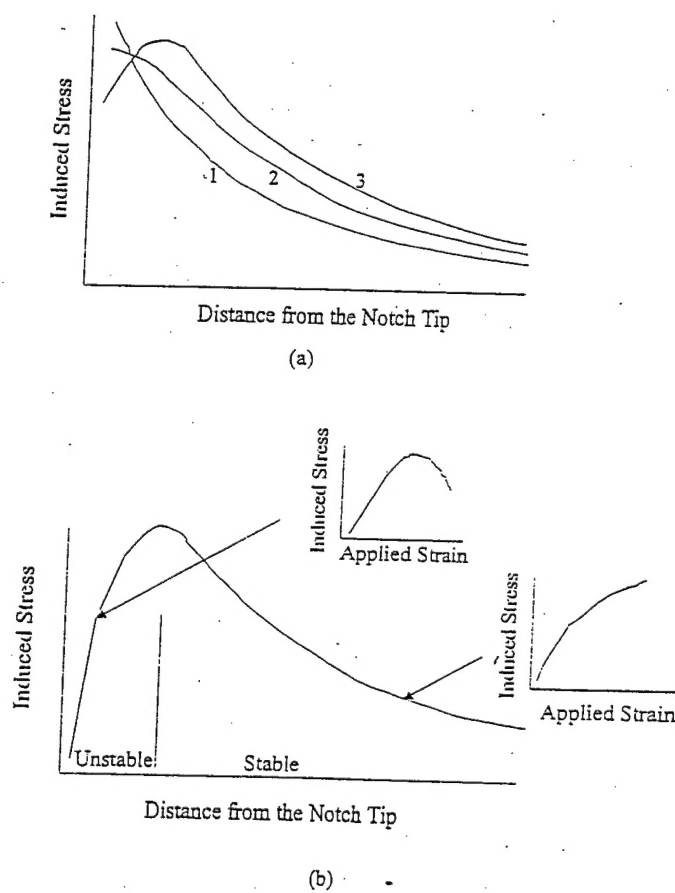


Figure 3. Stress Distribution along the Minimum Section from the Notch Tip as a function of damage: (a) Damage Increases from Curve 1 to Curve 3, (b) Stable and Unstable Zones when Damage Saturates at the Notch Tip

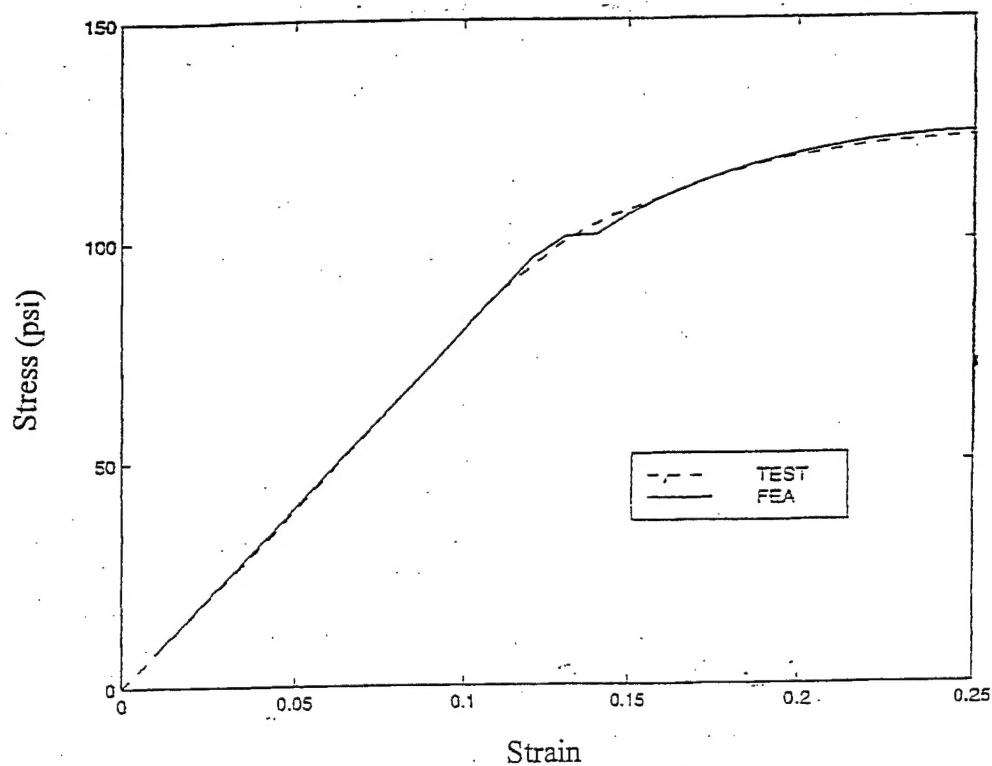


Figure 4. Stress-Strain Curves *set (have as you have)*

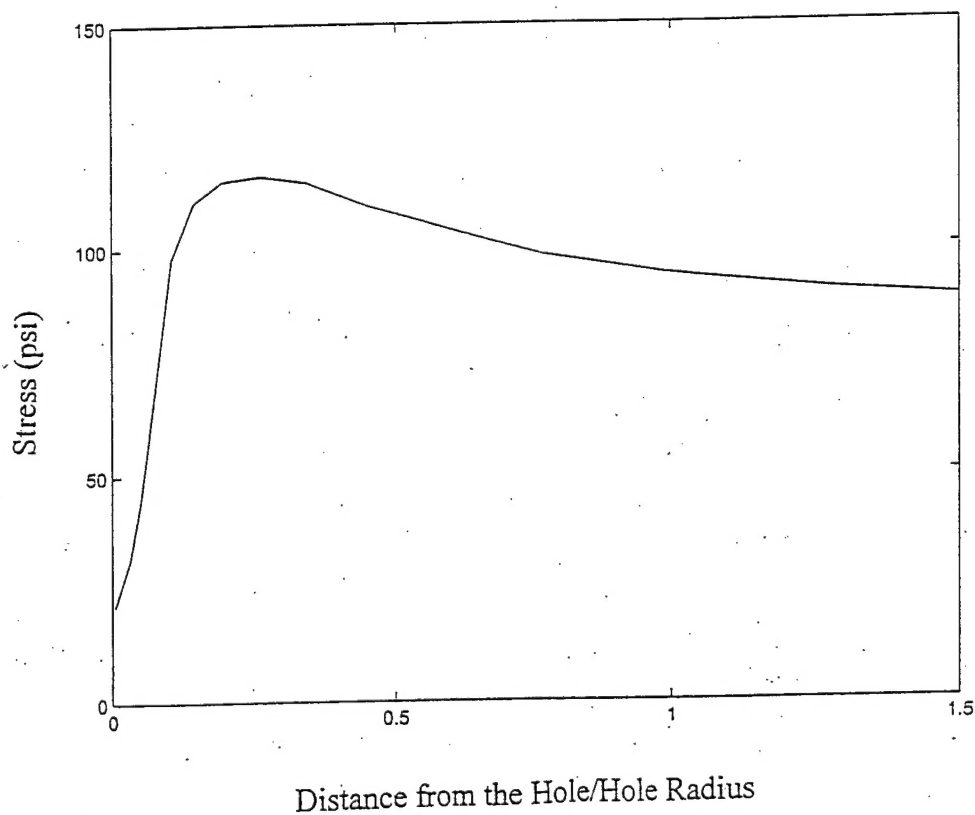


Figure 5. Normal Stress Distribution as a Function of the Normalized Distance from the Edge of the Hole (0.5 in. Hole Diameter)

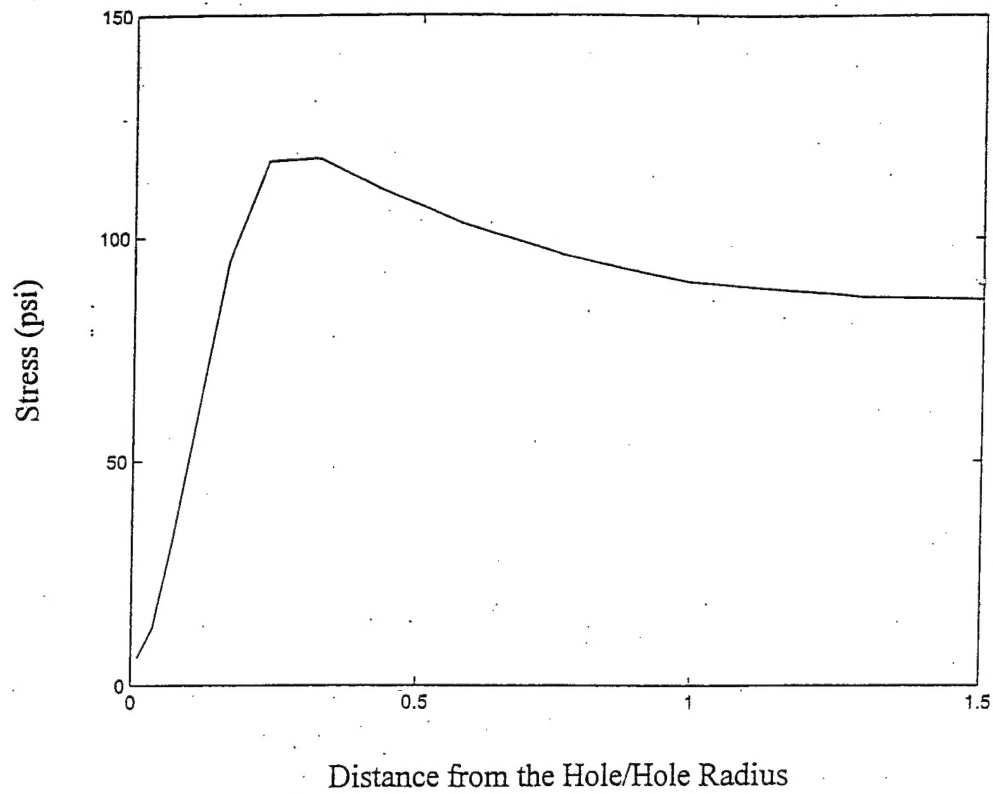


Figure 6. Normal Stress Distribution as a Function of the Normalized Distance from the Edge of the Hole (0.25 in. Hole Diameter)  $\chi$